UDC 539.3

BOUNDARY CONDITIONS IN THE VLASOV THEORY OF CYLINDRICAL SHELLS

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Semi-membrane theory of thin shells of Vlasov [1] reduces the number of the boundary conditions which have to be fulfilled at the curvilinear edges of the shell, to two. The remaining two conditions hold by virtue of the arbitrariness of the simple edge effect. In analogy with the membrane theory, when the shell is computed using the Vlasov theory, the tangential conditions are usually made to hold at the curvilinear edges and the discrepancies appearing in the nontangential conditions are removed with help of the simple edge effect.

In the present paper the boundary conditions at the curvilinear edges were obtained for the Vlasov system of equations using the asymptotic method. As a result, it was found that for certain types of clamping of the shell edges the boundary conditions sought must contain the non-tangential terms. The results obtained were confirmed by a numerical example.

Gol'denviezer generalized the Vlasov theory to embrace arbitary shells of zero curvature. The stress-strain state under consideration is called in [2], the generalized edge effect. The terminology and notation used below are those of [2].

1. Let the quantities sought have the variability index θ along the generatrices (which are assumed to coincide with the α-lines) and the variability index t along the directrices (which are the β-lines). Then we can replace α and β by the new variables ξ and η in the following manner:

$$\alpha = Rh_*^{\theta}\xi, \quad \beta = Rh_*^{t}\eta \tag{1.1}$$

where h_* denotes the relative half-thickness of the shell and R its characteristic radius.

In what follows we shall assume that the differentiation with respect to ξ and η does not appreciably increase or decrease the values of the functions sought.

It was shown in [2], that the quantities t and θ can assume the positive, as well as the negative values. When $t = 0$, the generalized edge effect loses completely its ability to decay, and we say that it degenerates. The variability indices t and θ are connected by the following relation:

$$t = {}^1/_4 + {}^1/_2\theta \quad (0 \leqslant t < {}^1/_2) \tag{1.2}$$

The stress-strain state in question has the following asymptotic representation (which is given in a somewhat different form in [2]):

$$u = h_*^{-1+\theta}u_*, \quad v = h_*^{-1+2\theta-t}v_*, \quad w = h_*^{-1+2\theta-2t}w_* \tag{1.3}$$

$$T_1 = h_*^{\circ} T_{1*}, \quad T_2 = h_*^{1-2t} T_{2*}, \quad S = h_*^{1/2-t} S_*$$
$$(G_1, G_2) = h_*^1 (G_{1*}, G_{2*}), \quad H = h_*^{1+t-\theta} H_*$$
$$N_1 = h_*^{3/2-2t} N_{1*}, \quad N_2 = h_*^{1-t} N_{2*}$$

Here and henceforth we assume that all quantities accompanied by an asterisk are of the same order.

Substituting (1. 3) and (1. 1) into the equations of cylindrical shells and restricting ourselves to the terms of order ε where

$$\varepsilon = O\left(h_*^{1-2t}\right) \tag{1. 4}$$

we obtain the Vlasov equations.

Let us write, with the help of the asymptotics (1. 3), only those equations which will be needed in computing the boundary conditions. They are

$$T_{1*} - h_*^{1-2t} \nu T_{2*} = 2E \frac{\partial u_*}{\partial \xi} \tag{1. 5}$$
$$h_*^{2-4t} T_{2*} - h_*^{1-2t} \nu T_{1*} = 2E \left(\frac{1}{B} \frac{\partial v_*}{\partial \eta} - R \frac{w_*}{R_2}\right)$$

2. Let uas assume that the internal state of stress of the shell can be separated into the generalized and the simple edge effect. Following [2], we shall write all unknown quantities as sums of two terms

$$P = P^{(g)} + h_*^{a} P^{(s)} \tag{2. 1}$$

Here P denotes any of the quantities (displacements, forces, moments) determining the stress-strain state of the shell. The superscripts (g) and (s) indicate whether the quantity in question belongs to the generalized, or the simple edge effect, respectively. We assume that $P^{(g)}$ can be constructed from the inhomogeneous equations of the generalized edge effect, and $P^{(s)}$ from the homogeneous equations of the simple edge effect, and $P^{(s)}$ is accompanied, for this reason, by a scale multiplier h_*^{a} where a is a number common to all quantities and chosen in accordance with the boundary conditions. As was shown in [2], the quantities of the simple edge effect have the following asymptotics:

$$u = h_*^{1/2} u_*, \quad v = h_*^{1-t} v_*, \quad w = h_*^{\circ} w_*, \quad \gamma_1 = h_*^{-1/2} \gamma_{1*} \tag{2. 2}$$
$$T_1 = h_*^{2-2t} T_{1*}, \quad T_2 = h_*^1 T_{2*}, \quad S = h_*^{3/2-t} S_*, \quad G_1 = h_*^2 G_{1*}$$
$$N_1 = h_*^{3/2} N_{1*}, \quad N_2 = h_*^{2-t} N_{2*}, \quad H = h_*^{5/2-t} H_*$$

Next we consider various boundary conditions.

H i n g e d e d g e. The boundary conditions at the edge $\alpha = \alpha_0$ have the form

$$T_1 = 0, \quad v = 0, \quad w = 0, \quad G_1 = 0$$

Taking into account (2. 1), (2. 2) and (1. 3), we can write these conditions in the form

$$h_*^{-1+2\theta-t} v_*^{(g)} + h_*^{a+1-t} v_*^{(s)} = 0, \quad h_*^0 T_{1*}^{(g)} + h_*^{a+2-2t} T_{1*}^{(s)} = 0 \tag{2. 3}$$
$$h_*^{-2+2t} w_*^{(g)} + h_*^{a} w_*^{(s)} = 0, \quad h_*^1 G_{1*}^{(g)} + h_*^{a+2} G_{1*}^{(s)} = 0$$

Putting $a = -2t$, we obtain a non-contradictory boundary value problem and the first two formulas of (2. 3) will then yield, with the accuracy defined by (1. 4), the following boundary conditions for the generalized edge effect

$$v^{(g)} = 0, \quad T_1^{(g)} = 0 \tag{2.4}$$

From (2.3) and (1.5) it follows that the displacement $v_*^{(g)}$ and hence $\partial v_*^{(g)}/\partial\eta$, $T_{1*}^{(g)}$ and $\varepsilon_{1*}^{(g)}$, are equal to zero at the edge to within some degree of accuracy. In this case we find from (1.5) and the last two formulas of (2.3), that the values of $w^{(g)}$ and $G_1^{(g)}$ are reduced at the edge in the following manner: $h_*^{-2+2t} w_*^{(g)}$ becomes $h_*^{-2t} w_*^{(g)}$, and $h_*^{1} G_{1*}^{(g)}$ becomes $h_*^{2-2t} G_{1*}^{(g)}$. As a result, the boundary conditions for the simple edge effect can be written as follows:

$$w^{(s)} = R_2 T_2^{(g)}/(2Eh), \quad G_1^{(s)} = -G_1^{(g)} \tag{2.5}$$

We note that the boundary conditions were derived with help of the complete equations of cylindrical shells.

In the case of a clamped edge just discussed, the principal stresses of the simple edge effect are h_*^{-1+2t} times smaller than the principal stresses of the generalized edge effect. This agrees with the presently held opinion that the simple edge effect is small near the hinged edge.

Let us consider another form of the boundary conditions describing a hinged support at the edge $\alpha = \alpha_0$

$$S = 0, \quad u = 0, \quad w = 0, \quad G_1 = 0 \tag{2.6}$$

We set $a = -1$. Following the procedure of the previous case, we arrive at the boundary conditions for the generalized edge effect, and the boundary conditions for the simple edge effect

$$u^{(g)} = 0, \quad w^{(g)} = 0 \tag{2.7}$$

$$S^{(s)} = -S^{(g)}, \quad G_1^{(s)} = 0 \tag{2.8}$$

The intensity of the simple edge effect in terms of the principal stresses is the same, as that of the generalized edge effect.

We note that in the case (2.6) of clamping, the boundary conditions do not separate in the usual manner. The conditions (2.7) for the generalized edge effect now contain together with the tangential displacement $u^{(g)}$, a non-tangential displacement $w^{(g)}$ and the conditions (2.8) for the simple edge effect contain also the tangential stress $S^{(s)}$.

F r e e e d g e $(\alpha = \alpha_0)$

$$S = 0, \quad T_1 = 0, \quad G_1 = 0, \quad N_1 = 0$$

Putting in (2.1) $a = -1$, we obtain the tangential conditions for the generalized edge effect and non-tangential conditions for the simple edge effect

$$T_1^{(g)} = 0, \quad S^{(g)} = 0 \tag{2.9}$$

$$G_1^{(s)} = -G_1^{(g)}, \quad N_1^{(s)} = 0 \tag{2.10}$$

C l a m p e d e d g e $(\alpha = \alpha_{10})$

$$u = 0, \quad v = 0, \quad w = 0, \quad \gamma_1 = 0$$

or, with the asymptotics taken into account,

$$h_*^{-1+\theta} u_*^{(g)} + h_*^{a+1/2} u_*^{(s)} = 0, \quad h_*^{-1+2\theta-t} v_*^{(g)} + h_*^{a+1-t} v_*^{(s)} = 0$$

$$h_*^{-1+2\theta-2t} w_*^{(g)} + h_*^{a} w_*^{(s)} = 0, \quad h_*^{-2+t} \gamma_{1*}^{(g)} + h_*^{a-1/2} \gamma_{1*}^{(s)} = 0$$

Let us put $a = -3/2 + t$. This yields the conditions for the generalized and the simple edge effect with the accuracy of up to the terms of order $O(h_*^{1/2-t})$

$$u^{(g)} = 0, \quad v^{(g)} = 0$$
$$w^{(s)} = 0, \quad \gamma_1^{(s)} = -\gamma_1^{(g)} \tag{2.11}$$

The accuracy of the conditions (2.11) is less than that of the equations of the generalized edge effect and the boundary conditions (2.4), (2.7) and (2.9), i.e. less than (1.4). It can however be increased to $O(h_*^{1-2t})$ using the method given in [2] on p. 303. Simple manipulations consisting of algebraic elimination of the quantities belonging to the simple edge effect from the complete boundary conditions, yields the following conditions for the generalized edge effect with the accuracy defined by (1.4)

$$u^{(g)} - \frac{\nu h}{\sqrt{3(1-\nu^2)}}\gamma_1^{(g)} = 0, \quad v^{(g)} = 0$$

and these conditions cannot, in general, be regarded as tangential.

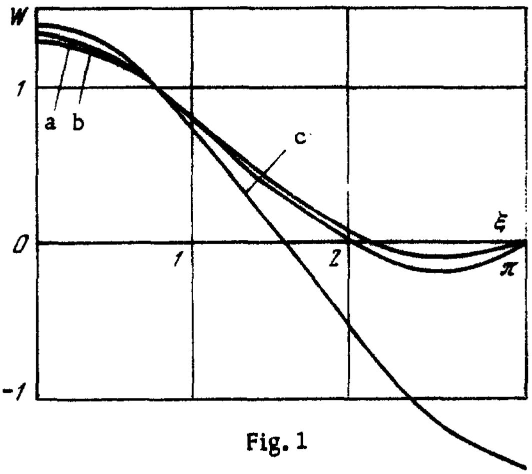

Fig. 1

We have investigated above four types of clamping of the shell edge. We have found that in two cases, for a hinged (2.6) and a clamped edge, the boundary conditions do not separate in the usual manner. We have omitted numerous variants of the boundary conditions describing elastic clamping of the edge and various types of hinged support. It can be expected, that the separation of the boundary conditions into parts will yield, in some of these cases, new results.

N o t e. *It can be shown that the separation of the boundary conditions obtained for* the cylindrical shells remains valid for any shell of zero curvature.

3. Let us perform a numerical computation of the state of stress of a circular cylindrical shell hinged along the edges (2.6), with relative thickness of $2h/R = 0.01$ and length $2l = 2\pi R$, acted upon by a normal load $p_n = q\cos 5\beta\cos\xi$. The coordinate ξ is directed along the axis of the cylinder $-l/R \leqslant \xi \leqslant l/R$, β is the circular coordinate and $\nu = 0.3$.

Let us determine the flexure of this shell: (a) using the classical theory, (b) using the theory of generalized edge effect with boundary conditions (2.7), (c) using the latter theory with tangential boundary conditions $u^{(g)} = 0$, $S^{(g)} = 0$. The results are presented in the form of graphs showing the dependence of $W = [wE/rq]\cdot 10^{-4}$ on ξ (see Fig. 1). The graphs show that the curve b describes the deflections satisfactorily, while the curve c gives a completely false picture of the deflections.

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